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FUNDAMENTAL SOLUTIONS OF THE THEORY OF UNIDIRECTIONAL COMPOSITES

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In the present study, we solve a problem concerning the action of a concentrated force in an infinite undirectional composite for the two- and three-dimensional cases. The approach that is taken makes it possible to express the solution of the problem of deformation by body forces in the form of series and integrals, solve the problem of a loaded half-space (half-plane), and asymptotically obtain known solutions on fiber rupture that can be employed in crack problems.

1. We will examine an infinite unidirectional composite in which the fibers form a square grid in the section perpendicular to the reinforcement direction z. The period of the grid is H + h (the cross-sectional area of the fibers is h^2). The numbers of the nodes are represented by the subscripts j and k. The dimensionless coordinates along the fibers $\xi = z/\sqrt{Hh}$, while the dimensionless displacement $w_{j,k} = u_{j,k}/\sqrt{Hh}$. The displacement satisfies the equation [1]

$$\partial^2 w_{j,k}/\partial\xi^2 + \beta^2\Delta_{jk}w = 0, \quad -\infty < j,\ k < \infty,$$
$$\Delta_{jk}w = w_{j-1,k} + w_{j,k-1} - 4w_{j,k} + w_{j+1,k} + w_{j,k+1} \tag{1.1}$$

and the auxiliary conditions

$$\sigma_{j,k} \to 0,\ |\xi| \to \infty,\ \sigma_{j,k} = E dw_{j,k}/d\xi; \tag{1.2}$$

$$[\sigma_z]|_{\xi=0} = E\frac{dw_{00}}{d\xi}\Big|_{\xi=+0} - E\frac{dw_{00}}{d\xi}\Big|_{\xi=-0} = 2Q. \tag{1.3}$$

Condition (1.3) gives a jump in the normal stress in the fiber k = j = 0 at the point $\xi = 0$. This corresponds to the application of a concentrated force $-2Qh^2$ to the fiber. Here, $\beta^2 = G/E$: E and G are the Young's modulus and shear modulus for the fiber and the binder; h and H are the width of the fiber and binder. The solution will be sought by means of double discrete Fourier transformation. Each equation of system (1.1) is multiplied by $\exp(-ijs) \times \exp(-ikq)$, $-\pi \le s, q \le \pi$ and summed over j, k within infinite limits. After completing some elementary transformations, we arrive at a linear differential equation with constant coefficients. The equation is of the second order in ξ relative to the double Fourier series:

$$w^{FF} = \sum_{j,k=-\infty}^{\infty} w_{j,k}(\xi)\exp(-ijs)\exp(-ikq). \tag{1.4}$$

The solution of (1.4) depends on two arbitrary constants. Although being independent of ξ, these constants generally depend on the parameters s and q. One of them is equal to zero, thanks to (1.2). After solving (1.4), we find $w_{j,k}$ from the inversion formula expressing

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the coefficients of Fourier series (1.4) in terms of its sum:

$$w_{j,k}(\xi) = \frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} c(s,q)\exp(-2\beta|\xi|R(s/2,q/2))\exp(-isj)\exp(-iqk)ds\,dq \qquad (1.5)$$

$$(R(s,\ q) = \sqrt{\sin^2 s + \sin^2 q}).$$

We now differentiate (1.5) at $\xi \to \pm 0$. The limiting values of the derivative differ only in sign, due to the appearance of the multiplier $d|\xi|/d\xi$. They are equal due to the continuity of the stresses at $\xi = 0$, $(j, k) \neq (0, 0)$. This means that

$$\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} c(s,q)R(s/2,q/2)\exp(-ijs)\exp(-ikq)\,ds\,dq = 0, \quad (j,k)\neq(0,0). \qquad (1.6)$$

Since all of the Fourier coefficients (1.6) (except for the zeroth coefficient) are equal to zero, the function $c(s, q)R(s/2, q/2)$ is a constant. The latter is found using condition (1.3). We obtain the displacement for the space from the action of the concentrated force

$$w_{j,k}(\xi) = \frac{-Q}{8\pi^2\beta E}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}[\exp(-2\beta|\xi|R(s/2,q/2))/R(s/2,q/2)]\times \qquad (1.7)$$
$$\times\exp(-isj)\exp(-iqk)\,ds\,dq.$$

Displacement (1.7) approaches zero at $j, k \to \infty$, since it is a Fourier coefficient of the function being summed. However, it does not approach zero too quickly, since the series of the squares of the displacements diverges. To prove this, we will use Parseval's identity and assume that the integrand in (1.7) is not quadratically summable. Thus, it has a singularity of the order $1/\sqrt{s^2 + q^2}$ at zero. This solution will also be the solution of the problem of the half-space $\xi > 0$ loaded on the surface by a force which creates a stress Q in the fiber $j = k = 0$ at $\xi = 0$. To be certain of this, we divide the force into two equal parts applied to the upper and lower half-spaces and we assume that the stresses at $\xi = 0$ are equal to zero everywhere except for the point (0, 0) (see (1.6)). We use Eq. (1.2) to find the stresses

$$\sigma_{j,k}(\xi) = \frac{Q\,\mathrm{sign}\,(\xi)}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\exp(-2\beta|\xi|R(s/2,q/2))\exp(-ijs)\exp(-iqk)ds\,dq. \qquad (1.8)$$

The following results were obtained from numerical realization of Eqs. (1.7-1.8). Table 1 shows values of $E\beta w_{j,k}(0)/2Q$ at nodes adjacent to the point of application of the concentrated force. The solution is symmetrical for negative j and k. Figure 1 shows the dependence of the stress distribution along the fibers on the action of the concentrated force (lines 1-5 correspond to j = 0; 0; 1; 0; 2 and k = 0; 1; 1; 2; 2). Let us now proceed to the two-dimensional case. Here, all of the functions depend on one integral variable j, the expression $w_{j,k+1} - 2w_{j,k} + w_{j,k-1}$ vanishes, and instead of (1.1) we have

$$d^2w_j/d\xi^2 + \beta^2(w_{j+1} - 2w_j + w_{j-1}) = 0, \qquad (1.9)$$
$$-\infty < j < \infty.$$

Proceeding similarly in the three-dimensional case, instead of (1.5) we obtain the expression

$$w_j(\xi) = \frac{1}{2\pi}\int_{-\pi}^{\pi} c(s)\exp(-2\beta|\xi||\sin(s/2)|)\exp(-ijs)\,ds. \qquad (1.10)$$

Finding the function $c(s) = -Q/2\beta E|\sin(s/2)|$, we see that integral (1.10) diverges at zero. It followed from subsequent analysis that the integral diverges due to formal application of the discrete Fourier transform. The quantity w_j does not vanish at $j \to \infty$ in the plane

TABLE 1

k	j: 0	1	2
	$E\beta w_{j,k}(0)/2Q$		
0	—0,3221240	—0,0826046	—0,0419314
1	—0,0826046	—0,0532128	—0,0355773
2	—0,0419314	—0,0355773	—0,0282115

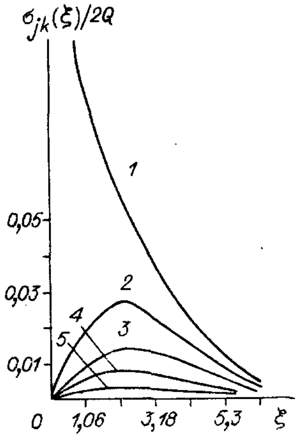

Fig. 1

case, so that the necessary condition for convergence of the series w^F is violated. To circumvent this obstacle, we need to examine a formulation of the problem in which the solutions will decrease sufficiently rapidly at $j \to \infty$. We then take the limit to obtain the solution of the initial problem. Here, instead of the problem of a half-plane loaded by a force at the boundary, we solve the analogous problem of a strip $0 \le \xi \le N$. At the boundary of the strip $\xi = N$, we distribute a stress which balances the load applied at the point $\xi = 0$, $j = 0$. We then take the limit $N \to \infty$. In this case, the balancing load approaches zero as $1/(2N + 1)$ but occupies the larger region $|j| \le N$, thus maintaining the balance condition. With finite N, after discrete Fourier transformation of Eq. (1.9), we arrive at a problem in the strip $0 \le \xi \le N$:

$$d^2w^F/d\xi^2 - 4\beta^2 \sin^2(s/2)w^F = 0,$$
$$E\frac{dw^F}{d\xi}\Big|_{\xi=0} = Q,\ E\frac{dw^F}{d\xi}\Big|_{\xi=N} = \frac{Q}{2N+1}\sum_{j=-N}^{N}\exp(ijs).$$

Having solved this, we find

$$w_j(\xi) = \frac{Q}{2\pi\beta E}\int_{-\pi}^{\pi}\frac{\frac{\sin(N+1/2)s}{(2N+1)\sin(s/2)}(\operatorname{ch}(\xi s)\exp(-ijs) - \operatorname{ch} s) - \operatorname{ch}(N-\xi)s\exp(-ijs) + \operatorname{ch}(N-1)s}{|\sin(s/2)|\operatorname{sh}(Ns)}ds \tag{1.11}$$

$(s = 2\beta|\sin(S/2)|)$. The integrand in (1.11) is bounded at 0. At $N \to \infty$, we have the solution for the half-plane in the form of a convergent integral

$$w_j(\xi) = \frac{Q}{8\pi\beta E}\int_{-\pi}^{\pi}\frac{\exp(-2\beta|\sin(s/2)|) - \exp(-2\beta|\xi|\sin(s/2)|)\exp(-ijs)}{|\sin(s/2)|}ds. \tag{1.12}$$

Displacement (1.12) vanishes at $\xi = 1$ and, in contrast to the three-dimensional solution (1.7), does not approach zero at $j, k \to \infty$. The corresponding stresses take the form

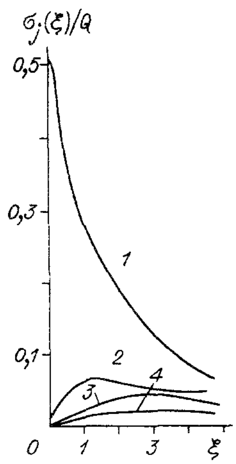

Fig. 2

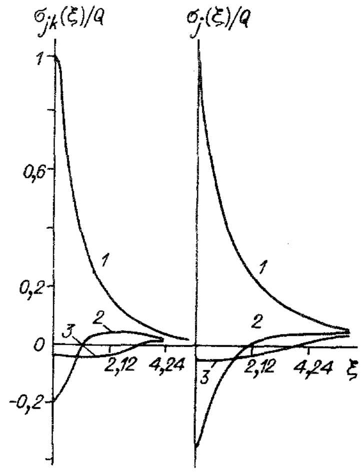

Fig. 3

$$\sigma_j(\xi) = \frac{Q\,\mathrm{sign}\,(\xi)}{4\pi}\int\limits_{-\pi}^{\pi}\exp\left(-2\beta\,|\sin(s/2)|\,|\xi|\right)\exp(-ijs)\,ds. \tag{1.13}$$

Let us return to the three-dimensional case and attempt to find the plane solution sought through the superposition of solutions (1.7) for concentrated forces applied at the boundary of a half-space at the points of the straight line j = 0. Each force creates the stress Q in the fiber to which it is applied. We substitute k − m for k in (1.7) and sum the integrand over m in infinite limits. Using the expression [2]

$$\sum_{m=-\infty}^{\infty}\exp(iqm) = 2\pi\sum_{n=-\infty}^{\infty}\delta(q-2\pi n) \tag{1.14}$$

(δ is the Dirac delta function), we again arrive at divergent integral (1.10). The reason for the divergence is the slow decrease of displacement (1.7) at infinity. In order to obtain Eq. (1.12), we recall that the displacement is determined only to within a constant shift along ξ. Thus, with summation of each finite sum over m, we will require that, as in (1.12) the displacement vanish at $\xi = 1$, $j = 0$, $k = 0$. Subtracting the corresponding term − which is independent of ξ, j but dependent on N − we again arrive at (1.12)

$$w_j(\xi) = \lim_{N\to\infty}\sum_{m=-N}^{N}\left(w_{j,k-m}(\xi) - w_{0,-m}(1)\right) =$$
$$= \frac{Q}{8\pi\beta E}\int\limits_{-\pi}^{\pi}\frac{\exp(-2\beta\,|\sin(s/2)|) - \exp(-2\beta\,|\xi|\,|\sin(s/2)|)\exp(-isj)\,ds}{|\sin(s/2)|}.$$

The process by which we obtain (1.12) can be regarded as a regularization of divergent integral (1.10).

Figure 2 shows the stress distibution along fibers with j = 0, 1, 2, 3 (lines 1-4). Here, a concentrated force $-2Qh^2$ is applied to the fiber j = k = 0 at the point $\xi = 0$. The displacements of the fibers at $\xi = 0$ are shown in Table 2.

2. By passing to the limit, we can use the solution of the problem of a concentrated force to obtain the solution for fiber rupture. This approach is similar to that which leads to the field of a dipole from the fields of two opposite charges. We will apply concentrated forces in opposite directions at the points $\xi = \pm\varepsilon$ of a fiber j = k = 0. We bring the forces closer together while having their magnitudes approach infinity. Thus, the "dipole moment" (the product of the force and distance) remains constant. As a result, we obtain the following from (1.7)

$$\lim_{\varepsilon\to 0}\frac{-Q2\varepsilon}{8\beta E\pi^2}\,\frac{I_{j,k}(\varepsilon) - I_{j,k}(-\varepsilon)}{2\varepsilon} = M\frac{dI}{d\xi}(0)$$

TABLE 2

j	$w_j(0)$	j	$w_j(0)$
0	—0,179442	2	0,244983
1	0,138862	3	0,308633

TABLE 3

$l=p=5$			$l=p=9$			$l=p=41$		
$\sigma_{j,k}(0)/Q$								
1,000	—0,153	—0,023	1,000	—0,147	—0,015	1,000	—0,146	—0,014
—0,153	—0,031	—0,015	—0,147	—0,025	—0,008	—0,146	—0,024	—0,007
—0,023	—0,015	—0,014	—0,015	—0,008	—0,004	—0,014	—0,007	—0,003

($I_{j,k}(\xi)$ is the double integral from (1.7)). We choose the constant M on the basis of the condition that the stress in the fiber j = k = 0 is equal to Q. We arrive at the solution

$$w_{j,k}(\xi)=\frac{-Q\,\mathrm{sign}\,(\xi)}{2\beta E}\frac{\int\limits_{-\pi}^{\pi}\int\limits_{-\pi}^{\pi}\exp(-2\beta|\xi|R(s/2,q/2))\exp(-isj)\exp(-iqk)ds\,dq}{\int\limits_{-\pi}^{\pi}\int\limits_{-\pi}^{\pi}R(s/2,q/2)\,ds\,dq}; \tag{2.1}$$

$$\sigma_{j,k}(\xi)=Q\int\limits_{-\pi}^{\pi}\int\limits_{-\pi}^{\pi}\exp(-2\beta|\xi|R(s/2,q/2))\exp(-ijs)\exp(-ikq)\times R(s/2,q/2)\,ds\,dq\Bigg/\int\limits_{-\pi}^{\pi}\int\limits_{-\pi}^{\pi}R(s/2,q/2)\,ds\,dq. \tag{2.2}$$

At $\xi = 0$, displacements (2.1) vanish everywhere except in the fiber j = k = 0. In the fiber, they undergo a discontinuity

$$w_{00}(+0)-w_{00}(-0)=-4\pi^2Q\Bigg/\left(\beta E\int\limits_{-\pi}^{\pi}\int\limits_{-\pi}^{\pi}R(s/2,q/2)\,ds\,dq\right).$$

The same action in the plane case leads to the solution

$$w_j(\xi)=-\frac{Q\,\mathrm{sign}\,(\xi)}{8\beta E}\int\limits_{-\pi}^{\pi}\exp(-2\beta|\xi||\sin(s/2)|)\exp(-ijs)\,ds,$$

$$\sigma_j(\xi)=\frac{Q}{4}\int\limits_{-\pi}^{\pi}\exp(-2\beta|\xi||\sin(s/2)|)\exp(-ijs)|\sin(s/2)|\,ds.$$

The displacement jump is equal to $-\pi Q/2\beta E$. Solution (2.1-2.2) was obtained previously by a different method [3]. Figure 3 shows curves of the stress change along the fiber with different k, j in the three-dimensional (lines 1-3 corresponding to j = 0; 0; 1 and k = 0; 1; 1) and two-dimensional (lines 1-3 corresponding to j = 0; 1; 2) cases.

3. Let us examine several special distributions of ruptures in the three-dimensional case. If there are several ruptures in a composite, then the superposition principle can be used to calculate the stress field.

A linear combination of the fields of the individual ruptures will obviously yield a stress state that satisfies the equilibrium equations. The coefficients of the linear combination are determined from the condition that the stresses at each rupture be equal to Q.

A. At $\xi = 0$, let the ruptures of the fibers form a rectangular grid with the periods p, ℓ. Since the ruptures take place under the same conditions, all of the coefficients of the

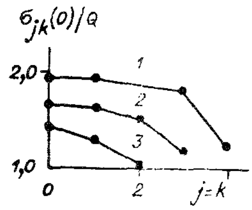

Fig. 4

linear combination will be equal and the infinite system will become one equation. Thanks to (1.14), the integrals in (2.3) reduce to finite sums. Let us determine the stress distribution in the plane $\xi = 0$:

$$\sigma_{j,k}(0) = Q\sum R(\pi k'/l, \pi j'/p)\cos(2\pi kk'/l)\cos(2\pi jj'/p)/\sum R(\pi k'/l, \pi j'/p) \tag{3.1}$$

(summation is carried out within the limits $|k'| \leq [\ell/2]$, $|j'| \leq [p/2]$, the brackets denoting the integral part of the number). Table 3 shows values of $\sigma_{j,k}(0)/Q$ (j, k = 0, 1, 2) calculated from Eq. (3.1). If the fibers are loaded by the stress $-Q$ at infinity and the stresses at the ruptures are equal to zero, then unity is subtracted from the numbers in Table 3. The effects of the defects on one another turn out to be negligible. The maximum stress concentration is seen in the fibers closest to a rupture. For the period $p = \ell = 9$, the overload nearly coincides with the situation of a double rupture and amounts to 14.7%.

B. Let the fiber ruptures be located at the nodes of a three-dimensional grid with the periods p, ℓ, L (L is the period along the fibers). The corresponding plane problem was examined in [4]. Proceeding as in case A, we obtain the stress

$$\frac{\sigma_{j,k}(\xi)}{Q} = \frac{\sum R(\pi k'/l, \pi j'/p)\cos(2\pi kk'/l)\cos(2\pi jj'/p)\sum\limits_{n=-\infty}^{\infty}\exp(-2\beta R(\pi k'/l, \pi j'/p)\,|\,nL-\xi\,|)}{\sum R(\pi k'/l, \pi j'/p)\,\mathrm{cth}\,(\beta LR(\pi k'/l, \pi j'/p))}$$

(summation is performed over k', j' as in (3.1)). The results of the calculations show that there is a shielding effect: the stresses decrease compared to the case $L = \infty$. The maximum overload on the fibers in the plane $\xi = 0$ is equal to 12% at $p = \ell = L = 5$ and 14% at $p = \ell = L = 9$.

C. If two ruptures are located at the points (0, 0, 0) and (ℓ, p, L), the stress at the point (k, j, ξ) has the form

$$\frac{\sigma_{j,k}(\xi)}{Q} = \frac{\int\limits_{-\pi}^{\pi}\int\limits_{-\pi}^{\pi} R(s/2, q/2)[\exp(-2\beta\,|\,\xi-L|R(s/2, q/2))\exp(isp)\exp(iql)+\exp(-2\beta|\xi|\,R(s/2, q/2))]\exp(-i(js+kq))\,ds\,dq}{\int\limits_{-\pi}^{\pi}\int\limits_{-\pi}^{\pi} R(s/2, q/2)[1+\exp(-2\beta\,|\,L\,|\,R(s/2, q/2))\exp(-isp)\exp(-ilq)]\,ds\,dq}.$$

If both ruptures lie in the same plane (L = 0) next to one another (p = 1, ℓ = 0), then the maximum stress is reached at the point (j, k) = (1, 1) and is equal to 0.2002. If the composite is subjected to tension at infinity, then the stress concentration reaches 1.2002. The most dangerous situation is when the location of the ruptures corresponds to p = 2, ℓ = 0. In the fiber lying between the ruptures, the stress concentration in tension at infinity is 1.29.

D. Let us now examine a square crack lying within the plane $\xi = 0$, $-N \leq j, k \leq N$. The composite is subjected to tension at infinity. The maximum stresses are reached on a continuation of the axes of symmetry of the square parallel to the sides at the points closest to the contour of the square. Figure 4 shows the stresses in the fibers directly adjacent to the side of a square crack for N = 3, 2, 1 (lines 1-3). For greater clarity, the points belonging

TABLE 4

k	j = 0	1	2	3
	$w_{j,k}(0)$			
0	—1,0160	1,0160	0,1724	0,0608
1	—0,1244	0,1244	0,0743	0,0415
2	—0,0263	0,0263	0,0294	0,0219

TABLE 5

Number of ruptured fibers	ν
1×1	1
3×3	2,66
5×5	4,24
7×7	5,8
9×9	7,3

TABLE 6

Number of ruptured fibers	ν
1	1
3	2,8
5	4,7
7	6,6
50	39,2

to each case are joined by straight lines. Since the stresses are symmetrical, we have shown only the right sides of the graphs. The maximum stress concentrations are equal to 1.98, 1.73, 1.46, and 1.15 at N = 3, 2, 1, and 0 (single rupture). The corresponding maximum dimensionless displacements, multiplied by $\beta E/2Q\pi^2$, are as follows: 2.16, 1.57, 0.99, 0.42.

4. Using the fundamental solutions in Part 1, we will solve the problem of a force couple formed by two equal (in terms of modulus) concentrated but oppositely directed forces applied to adjacent fibers at $\xi = 0$. This solution was obtained by a different method in [1]. We will examine the plane case first. Let a concentrated force causing stress jump 2Q in a fiber be applied at the point j = 0, $\xi = 0$, and let another concentrated force of the opposite sign be applied at j = 1, $\xi = 0$. This produces a moment of forces acting in the counterclockwise direction. Using (1.12-1.13), we obtain the displacements and stresses from the couple:

$$w_j(\xi) = \frac{Q}{\pi\beta E}\int_0^{\pi/2} \sin(2j-1)s \exp(-2\beta|\xi|\sin s)\,ds.$$

In particular, at $\xi = 0$, $w_j(0) = Q/\pi\beta E(2j-1)$, the displacement difference $\delta w_0 = w_1(0) - w_0(0) = 2Q/\pi\beta E$,

$$\sigma_j(\xi) = \frac{2Q\,\mathrm{sign}(\xi)}{\pi}\int_0^{\pi/2} \sin s \sin(1-2j)s \exp(-2\beta|\xi|\sin s)\,ds. \tag{4.1}$$

At $\xi = 0$, stresses (4.1) are equal to zero at all j except for j = 0, 1. We can use (1.7-1.8) to find the displacements and stresses in the three-dimensional case from the action of a pair of forces applied at the points j = 0, k = 0, and j = 1, k = 0:

$$w_{j,k}(\xi) = \frac{4Q}{\beta E\pi^2}\int_0^{\pi/2}\int_0^{\pi/2} [\cos(2kq)\sin s \sin(2j-1)s \exp(-2\beta|\xi|\times R(s,q))/R(s,q)]\,ds\,dq,$$

$$\sigma_{j,k}(\xi) = \frac{8Q\,\mathrm{sign}(\xi)}{\pi^2}\int_0^{\pi/2}\int_0^{\pi/2} \cos(2qk)\sin s \sin(1-2j)s \exp(-2\beta|\xi|R(s,q))\,ds\,dq.$$

As in the two-dimensional case, the stresses at $\xi = 0$ are nontrivial only in the fibers to which the forces are applied. Table 4 shows the displacements at $\xi = 0$ in the fibers closest to the couple. With negative values of the indices, the displacements are continued along the symmetry axis.

5. The system of equations which describes the displacements of the edges of a square crack in a composite (see D in Part 3) has one shortcoming: the conditionality of the system deteiorates with an increase in the dimensions of the crack. As an illustration, we determine the ratio ν of the maximum and minimum (with respect to modulus) eigenvalues of the matrix formed in the solution of the problem of a square normal-rupture crack loaded by a constant stress (see D in Part 3). Tables 5 and 6 show the results for the three- and two-dimensional cases. The conditionality number ν increases roughly in proportion to crack size.

This becomes particularly evident in the solution of the problem of an infinite layer loaded by a constant stress at the boundaries. If the ends of the fibers exceeding the boundary are regarded as points of rupture of fibers of equal intensity in an infinite medium, then we can sum the stresses from all of the elementary ruptures to obtain a zero stress at each point of the layer. This result becomes obvious if we recall that the solution for a single rupture is self-balanced. Thus, the stresses at the boundary of the layer are equal to zero, while the solution of the corresponding system (the intensity of the ruptures) is nontrivial. As a result, the system of equations becomes indeterminate with the transition to an infinite region. The author of [1] used fundamental solutions to obtain a system of integral and algebraic equations describing the stress state of a composite having fiber ruptures and binder delaminations. Here, the boundaries of the composite that were perpendicular to the fibers were regarded as fiber ruptures in an infinitely large specimen. In connection with the deterioration in the conditionality of the system with an increase in the number of equations, it is best to satify the conditions in stresses on the external boundaries perpendicular to the fibers by means of the fundamental solution on the concentrated force, while fundamental solution (2.1-2.2) is used to satisfy the conditions for the internal ruptures. As an illustration of this, let us again examine the problem of a uniformly loaded layer. Here, it will suffice to apply concentrated forces ensuring the required stress to the ends of each fiber which exceed the boundaries. As a result, the entire layer turns out to be balanced. In this case, the infinite system decomposes into separate unidimensional equations and ν turns out to have a value of one. In all of the numerical calculations, $\beta = \sqrt{2/9}$.

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